Question ID fc3d783a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: fc3d783a

3.1

In the xy-plane, a line with equation $2y = 4.5$ intersects a parabola at exactly one point. If the parabola has equation $y = -4x^2 + bx$, where b is a positive constant, what is the value of b?

ID: fc3d783a Answer

Correct Answer: 6

Rationale

The correct answer is 6. It's given that a line with equation $2y = 4.5$ intersects a parabola with equation $y = -4x^2 + bx$, where b is a positive constant, at exactly one point in the xy-plane. It follows that the system of equations consisting of $2y = 4.5$ and $y = -4x^2 + bx$ has exactly one solution. Dividing both sides of the equation of the line by 2 yields $y = 2.25$. Substituting 2.25 for y in the equation of the parabola yields $2.25 = -4x^2 + bx$. Adding $4x^2$ and subtracting bx from both sides of this equation yields $4x^2 - bx + 2.25 = 0$. A quadratic equation in the form of $ax^2 + bx + c = 0$, where a, b, and c are constants, has exactly one solution when the discriminant, $b^2 - 4ac$, is equal to zero. Substituting 4 for a and 2.25 for c in the expression $b^2 - 4ac$ and setting this expression equal to 0 yields $b^2 - 4(4)(2.25) = 0$, or $b^2 - 36 = 0$. Adding 36 to each side of this equation yields $b^2 = 36$. Taking the square root of each side of this equation yields $b = \pm 6$. It's given that b is positive, so the value of b is 6.

Question Difficulty: Hard

Question ID 4661e2a9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 4661e2a9

3.2

$x - y = 1$

$x + y = x^2 - 3$

Which ordered pair is a solution to the system of equations above?

A. $(1+\sqrt{3}, \sqrt{3})$

B. $(\sqrt{3}, -\sqrt{3})$

C. $(1+\sqrt{5}, \sqrt{5})$

D. $(\sqrt{5}, -1+\sqrt{5})$

ID: 4661e2a9 Answer

Correct Answer: A

Rationale

Choice A is correct. The solution to the given system of equations can be found by solving the first equation for x, which gives $x = y + 1$, and substituting that value of x into the second equation which gives $y + 1 + y = (y+1)^2 - 3$. Rewriting this equation by adding like terms and expanding $(y+1)^2$ gives $2y + 1 = y^2 + 2y - 2$. Subtracting $2y$ from both sides of this equation gives $1 = y^2 - 2$. Adding to 2 to both sides of this equation gives $3 = y^2$. Therefore, it follows that $y = \pm\sqrt{3}$. Substituting $\sqrt{3}$ for y in the first equation yields $x - \sqrt{3} = 1$. Adding $\sqrt{3}$ to both sides of this equation yields $x = 1 + \sqrt{3}$. Therefore, the ordered pair $(1+\sqrt{3}, \sqrt{3})$ is a solution to the given system of equations.

Choice B is incorrect. Substituting $\sqrt{3}$ for x and $-\sqrt{3}$ for y in the first equation yields $\sqrt{3} - (-\sqrt{3}) = 1$, or $2\sqrt{3} = 1$, which isn't a true statement. Choice C is incorrect. Substituting $1 + \sqrt{5}$ for x and $\sqrt{5}$ for y in the second equation yields $(1+\sqrt{5}) + \sqrt{5} = (1+\sqrt{5})^2 - 3$, or $1 + 2\sqrt{5} = 2\sqrt{5} + 3$, which isn't a true statement. Choice D is incorrect. Substituting $\sqrt{5}$ for x and $(-1+\sqrt{5})$ for y in the second equation yields $\sqrt{5} + (-1+\sqrt{5}) = (\sqrt{5})^2 - 3$, or $2\sqrt{5} - 1 = 2$, which isn't a true statement.

Question Difficulty: Hard

Question ID f65288e8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: f65288e8 3.3

$$\frac{1}{x^2+10x+25}=4$$

If x is a solution to the given equation, which of the following is a possible value of $x+5$?

A. $\frac{1}{2}$

B. $\frac{5}{2}$

C. $\frac{9}{2}$

D. $\frac{11}{2}$

ID: f65288e8 Answer

Correct Answer: A

Rationale

Choice A is correct. The given equation can be rewritten as $\frac{1}{(x+5)^2}=4$. Multiplying both sides of this equation by $(x+5)^2$ yields $1=4(x+5)^2$. Dividing both sides of this equation by 4 yields $\frac{1}{4}=(x+5)^2$. Taking the square root of both sides of this equation yields $\frac{1}{2}=x+5$ or $-\frac{1}{2}=x+5$. Therefore, a possible value of $x+5$ is $\frac{1}{2}$.

Choices B, C, and D are incorrect and may result from computational or conceptual errors.

Question Difficulty: Hard

Question ID f2f3fa00

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: f2f3fa00

3.4

During a 5-second time interval, the average acceleration a, in meters per second squared, of an object with an initial velocity of 12 meters per second is defined by the equation $a = \frac{v_f - 12}{5}$, where v_f is the final velocity of the object in meters per second. If the equation is rewritten in the form $v_f = xa + y$, where x and y are constants, what is the value of x ?

ID: f2f3fa00 Answer

Rationale

The correct answer is 5. The given equation can be rewritten in the form $v_f = xa + y$, like so:

$$a = \frac{v_f - 12}{5}$$

$$v_f - 12 = 5a$$

$$v_f = 5a + 12$$

It follows that the value of x is 5 and the value of y is 12.

Question Difficulty: Hard

Question ID 6ce95fc8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 6ce95fc8 3.5

$2x^2 - 2 = 2x + 3$

Which of the following is a solution to the equation above?

A. 2

B. $1 - \sqrt{11}$

C. $\frac{1}{2} + \sqrt{11}$

D. $\frac{1+\sqrt{11}}{2}$

ID: 6ce95fc8 Answer

Correct Answer: D

Rationale

Choice D is correct. A quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are constants, can be solved using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Subtracting $2x + 3$ from both sides of the given equation yields $2x^2 - 2x - 5 = 0$. Applying the quadratic formula, where $a = 2$, $b = -2$, and $c = -5$, yields $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-5)}}{2(2)}$. This can be rewritten as $x = \frac{2 \pm \sqrt{44}}{4}$. Since $\sqrt{44} = \sqrt{2^2(11)}$, or $2\sqrt{11}$, the equation can be rewritten as $x = \frac{2 \pm 2\sqrt{11}}{4}$. Dividing 2 from both the numerator and denominator yields $\frac{1+\sqrt{11}}{2}$ or $\frac{1-\sqrt{11}}{2}$. Of these two solutions, only $\frac{1+\sqrt{11}}{2}$ is present among the choices. Thus, the correct choice is D.

Choice A is incorrect and may result from a computational or conceptual error. Choice B is incorrect and may result from using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{a}$ instead of $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ as the quadratic formula. Choice C is incorrect and may result from rewriting $\sqrt{44}$ as $4\sqrt{11}$ instead of $2\sqrt{11}$.

Question Difficulty: Hard

Question ID c303ad23

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: c303ad23 3.6

If $3x^2 - 18x - 15 = 0$, what is the value of $x^2 - 6x$?

ID: c303ad23 Answer

Correct Answer: 5

Rationale

The correct answer is 5. Dividing each side of the given equation by 3 yields $x^2 - 6x - 5 = 0$. Adding 5 to each side of this equation yields $x^2 - 6x = 5$. Therefore, if $3x^2 - 18x - 15 = 0$, the value of $x^2 - 6x$ is 5.

Question Difficulty: Hard

Question ID 7bd10ef3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 7bd10ef3

3.7

$2x^2 - 4x = t$

In the equation above, t is a constant. If the equation has no real solutions, which of the following could be the value of t ?

A. -3

B. -1

C. 1

D. 3

ID: 7bd10ef3 Answer

Correct Answer: A

Rationale

Choice A is correct. The number of solutions to any quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are constants, can be found by evaluating the expression $b^2 - 4ac$, which is called the discriminant. If the value of $b^2 - 4ac$ is a positive number, then there will be exactly two real solutions to the equation. If the value of $b^2 - 4ac$ is zero, then there will be exactly one real solution to the equation. Finally, if the value of $b^2 - 4ac$ is negative, then there will be no real solutions to the equation.

The given equation $2x^2 - 4x = t$ is a quadratic equation in one variable, where t is a constant. Subtracting t from both sides of the equation gives $2x^2 - 4x - t = 0$. In this form, $a = 2$, $b = -4$, and $c = -t$. The values of t for which the equation has no real solutions are the same values of t for which the discriminant of this equation is a negative value. The discriminant is equal to $(-4)^2 - 4(2)(-t)$; therefore, $(-4)^2 - 4(2)(-t) < 0$. Simplifying the left side of the inequality gives $16 + 8t < 0$. Subtracting 16 from both sides of the inequality and then dividing both sides by 8 gives $t < -2$. Of the values given in the options, -3 is the only value that is less than -2. Therefore, choice A must be the correct answer.

Choices B, C, and D are incorrect and may result from a misconception about how to use the discriminant to determine the number of solutions of a quadratic equation in one variable.

Question Difficulty: Hard

Question ID 66bce0c1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 66bce0c1

3.8

$$\sqrt{2x+6}+4=x+3$$

What is the solution set of the equation above?

A. $\{-1\}$

B. $\{5\}$

C. $\{-1, 5\}$

D. $\{0, -1, 5\}$

ID: 66bce0c1 Answer

Correct Answer: B

Rationale

Choice B is correct. Subtracting 4 from both sides of $\sqrt{2x+6}+4=x+3$ isolates the radical expression on the left side of the equation as follows: $\sqrt{2x+6}=x-1$. Squaring both sides of $\sqrt{2x+6}=x-1$ yields $2x+6=x^2-2x+1$. This equation can be rewritten as a quadratic equation in standard form: $x^2-4x-5=0$. One way to solve this quadratic equation is to factor the expression x^2-4x-5 by identifying two numbers with a sum of -4 and a product of -5. These numbers are -5 and 1. So the quadratic equation can be factored as $(x-5)(x+1)=0$. It follows that 5 and -1 are the solutions to the quadratic equation. However, the solutions must be verified by checking whether 5 and -1 satisfy the original equation, $\sqrt{2x+6}+4=x+3$. When $x=-1$, the original equation gives $\sqrt{2(-1)+6}+4=(-1)+3$, or $6=2$, which is false. Therefore, -1 does not satisfy the original equation. When $x=5$, the original equation gives $\sqrt{2(5)+6}+4=5+3$, or $8=8$, which is true. Therefore, $x=5$ is the only solution to the original equation, and so the solution set is $\{5\}$.

Choices A, C, and D are incorrect because each of these sets contains at least one value that results in a false statement when substituted into the given equation. For instance, in choice D, when 0 is substituted for x into the given equation, the result is $\sqrt{2(0)+6}+4=(0)+3$, or $\sqrt{6}+4=3$. This is not a true statement, so 0 is not a solution to the given equation.

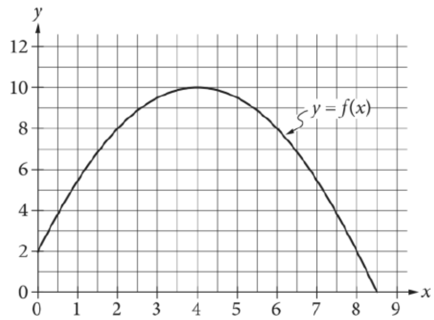
Question Difficulty: Hard

Question ID 97e50fa2

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 97e50fa2

3.9



The graph of the function *f*, defined by $f(x) = -\frac{1}{2}(x-4)^2 + 10$, is shown in the *xy*-plane above. If the function *g* (not shown) is defined by $g(x) = -x + 10$, what is one possible value of *a* such that $f(a) = g(a)$?

ID: 97e50fa2 Answer

Rationale

The correct answer is either 2 or 8. Substituting $x = a$ in the definitions for f and g gives $f(a) = -\frac{1}{2}(a-4)^2 + 10$ and $g(a) = -a + 10$, respectively. If $f(a) = g(a)$, then $-\frac{1}{2}(a-4)^2 + 10 = -a + 10$. Subtracting 10 from both sides of this equation gives $-\frac{1}{2}(a-4)^2 = -a$. Multiplying both sides by -2 gives $(a-4)^2 = 2a$. Expanding $(a-4)^2$ gives $a^2 - 8a + 16 = 2a$. Combining the like terms on one side of the equation gives $a^2 - 10a + 16 = 0$. One way to solve this equation is to factor $a^2 - 10a + 16$ by identifying two numbers with a sum of -10 and a product of 16. These numbers are -2 and -8, so the quadratic equation can be factored as $(a-2)(a-8) = 0$. Therefore, the possible values of a are either 2 or 8. Note that 2 and 8 are examples of ways to enter a correct answer.

Alternate approach: Graphically, the condition $f(a) = g(a)$ implies the graphs of the functions $y = f(x)$ and $y = g(x)$ intersect at $x = a$. The graph $y = f(x)$ is given, and the graph of $y = g(x)$ may be sketched as a line with y-intercept 10 and a slope of -1 (taking care to note the different scales on each axis). These two graphs intersect at $x = 2$ and $x = 8$.

Question Difficulty: Hard

Question ID 3d12b1e0

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 3d12b1e0

3.10

$$-16x^2 - 8x + c = 0$$

In the given equation, c is a constant. The equation has exactly one solution. What is the value of c?

ID: 3d12b1e0 Answer

Correct Answer: -1

Rationale

The correct answer is -1. A quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are constants, has exactly one solution when its discriminant, $b^2 - 4ac$, is equal to 0. In the given equation, $-16x^2 - 8x + c = 0$, $a = -16$ and $b = -8$. Substituting -16 for a and -8 for b in $b^2 - 4ac$ yields $(-8)^2 - 4(-16)(c)$, or $64 + 64c$. Since the given equation has exactly one solution, $64 + 64c = 0$. Subtracting 64 from both sides of this equation yields $64c = -64$. Dividing both sides of this equation by 64 yields $c = -1$. Therefore, the value of c is -1.

Question Difficulty: Hard

Question ID 71014fb1

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 71014fb1

3.11

$$(x-1)^2=-4$$

How many distinct real solutions does the given equation have?

A. Exactly one

B. Exactly two

C. Infinitely many

D. Zero

ID: 71014fb1 Answer

Correct Answer: D

Rationale

Choice D is correct. Any quantity that is positive or negative in value has a positive value when squared. Therefore, the left-hand side of the given equation is either positive or zero for any value of x. Since the right-hand side of the given equation is negative, there is no value of x for which the given equation is true. Thus, the number of distinct real solutions for the given equation is zero.

Choices A, B, and C are incorrect and may result from conceptual errors.

Question Difficulty: Hard

Question ID e9349667

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: e9349667

3.12

$$y = x^2 + 2x + 1$$
$$x + y + 1 = 0$$

If (x_1, y_1) and (x_2, y_2) are the two solutions to the system of equations above, what is the value of $y_1 + y_2$?

A. -3

B. -2

C. -1

D. 1

ID: e9349667 Answer

Correct Answer: D

Rationale

Choice D is correct. The system of equations can be solved using the substitution method. Solving the second equation for y gives $y = -x - 1$. Substituting the expression $-x - 1$ for y into the first equation gives $-x - 1 = x^2 + 2x + 1$. Adding $x + 1$ to both sides of the equation yields $x^2 + 3x + 2 = 0$. The left-hand side of the equation can be factored by finding two numbers whose sum is 3 and whose product is 2, which gives $(x + 2)(x + 1) = 0$. Setting each factor equal to 0 yields $x + 2 = 0$ and $x + 1 = 0$, and solving for x yields $x = -2$ or $x = -1$. These values of x can be substituted for x in the equation $y = -x - 1$ to find the corresponding y-values: $y = -(-2) - 1 = 2 - 1 = 1$ and $y = -(-1) - 1 = 1 - 1 = 0$. It follows that $(-2, 1)$ and $(-1, 0)$ are the solutions to the given system of equations. Therefore, $(x_1, y_1) = (-2, 1)$, $(x_2, y_2) = (-1, 0)$, and $y_1 + y_2 = 1 + 0 = 1$.

Choice A is incorrect. The solutions to the system of equations are $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (-1, 0)$. Therefore, -3 is the sum of the x-coordinates of the solutions, not the sum of the y-coordinates of the solutions. Choices B and C are incorrect and may be the result of computation or substitution errors.

Question Difficulty: Hard

Question ID b03adde3

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: b03adde3

3.13

If $u-3=\frac{6}{t-2}$, what is t in terms of u?

A. $t=\frac{1}{u}$

B. $t=\frac{2u+9}{u}$

C. $t=\frac{1}{u-3}$

D. $t=\frac{2u}{u-3}$

ID: b03adde3 Answer

Correct Answer: D

Rationale

Choice D is correct. Multiplying both sides of the given equation by $t-2$ yields $(t-2)(u-3)=6$. Dividing both sides of this equation by $u-3$ yields $t-2=\frac{6}{u-3}$. Adding 2 to both sides of this equation yields $t=\frac{6}{u-3}+2$, which can be rewritten as $t=\frac{6}{u-3}+\frac{2(u-3)}{u-3}$. Since the fractions on the right-hand side of this equation have a common denominator, adding the fractions yields $t=\frac{6+2(u-3)}{u-3}$. Applying the distributive property to the numerator on the right-hand side of this equation yields $t=\frac{6+2u-6}{u-3}$, which is equivalent to $t=\frac{2u}{u-3}$.

Choices A, B, and C are incorrect and may result from various misconceptions or miscalculations.

Question Difficulty: Hard

Question ID 30281058

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 30281058

3.14

In the *xy*-plane, the graph of $y = x^2 - 9$ intersects line *p* at $(1, a)$ and $(5, b)$, where *a* and *b* are constants. What is the slope of line *p* ?

A. 6

B. 2

C. -2

D. -6

ID: 30281058 Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that the graph of $y = x^2 - 9$ and line p intersect at $(1, a)$ and $(5, b)$. Therefore, the value of y when $x = 1$ is the value of a, and the value of y when $x = 5$ is the value of b. Substituting 1 for x in the given equation yields $y = (1)^2 - 9$, or $y = -8$. Similarly, substituting 5 for x in the given equation yields $y = (5)^2 - 9$, or $y = 16$. Therefore, the intersection points are $(1, -8)$ and $(5, 16)$. The slope of line p is the ratio of the change in y to the change in x between these two points: $\frac{16 - (-8)}{5 - 1} = \frac{24}{4}$, or 6.

Choices B, C, and D are incorrect and may result from conceptual or calculation errors in determining the values of a, b, or the slope of line p.

Question Difficulty: Hard

Question ID 5910bfff

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 5910bfff

3.15

$$D = T - \frac{9}{25}(100 - H)$$

The formula above can be used to approximate the dew point *D*, in degrees Fahrenheit, given the temperature *T*, in degrees Fahrenheit, and the relative humidity of *H* percent, where $H > 50$. Which of the following expresses the relative humidity in terms of the temperature and the dew point?

A. $H = \frac{25}{9}(D - T) + 100$

B. $H = \frac{25}{9}(D - T) - 100$

C. $H = \frac{25}{9}(D + T) + 100$

D. $H = \frac{25}{9}(D + T) - 100$

ID: 5910bfff Answer

Correct Answer: A

Rationale

Choice A is correct. It's given that $D = T - \frac{9}{25}(100 - H)$. Solving this formula for H expresses the relative humidity in terms of the temperature and the dew point. Subtracting T from both sides of this equation yields $D - T = -\frac{9}{25}(100 - H)$. Multiplying both sides by $-\frac{25}{9}$ yields $-\frac{25}{9}(D - T) = 100 - H$. Subtracting 100 from both sides yields $-\frac{25}{9}(D - T) - 100 = -H$. Multiplying both sides by -1 results in the formula $\frac{25}{9}(D - T) + 100 = H$.

Choices B, C, and D are incorrect and may result from errors made when rewriting the given formula.

Question Difficulty: Hard

Question ID 1697ffcf

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 1697ffcf

3.16

In the *xy*-plane, the graph of $y = 3x^2 - 14x$ intersects the graph of $y = x$ at the points $(0, 0)$ and (a, a). What is the value of *a* ?

ID: 1697ffcf Answer

Rationale

The correct answer is 5. The intersection points of the graphs of $y = 3x^2 - 14x$ and $y = x$ can be found by solving the system consisting of these two equations. To solve the system, substitute x for y in the first equation. This gives $x = 3x^2 - 14x$. Subtracting x from both sides of the equation gives $0 = 3x^2 - 15x$. Factoring $3x$ out of each term on the left-hand side of the equation gives $0 = 3x(x - 5)$. Therefore, the possible values for x are 0 and 5. Since $y = x$, the two intersection points are $(0,0)$ and $(5,5)$. Therefore, $a = 5$.

Question Difficulty: Hard

Question ID ff2e5c76

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: ff2e5c76

3.17

$$x^2 - 40x - 10 = 0$$

What is the sum of the solutions to the given equation?

A. 0

B. 5

C. 10

D. 40

ID: ff2e5c76 Answer

Correct Answer: D

Rationale

Choice D is correct. Adding 10 to each side of the given equation yields $x^2 - 40x = 10$. To complete the square, adding $\left(\frac{40}{2}\right)^2$, or 20^2, to each side of this equation yields $x^2 - 40x + 20^2 = 10 + 20^2$, or $(x - 20)^2 = 410$. Taking the square root of each side of this equation yields $x - 20 = \pm\sqrt{410}$. Adding 20 to each side of this equation yields $x = 20 \pm \sqrt{410}$. Therefore, the solutions to the given equation are $x = 20 + \sqrt{410}$ and $x = 20 - \sqrt{410}$. The sum of these solutions is $\left(20 + \sqrt{410}\right) + \left(20 - \sqrt{410}\right)$, or 40.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 2c5c22d0

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 2c5c22d0

3.18

$y = x^2 + 3x - 7$
$y - 5x + 8 = 0$

How many solutions are there to the system of equations above?

A. There are exactly 4 solutions.

B. There are exactly 2 solutions.

C. There is exactly 1 solution.

D. There are no solutions.

ID: 2c5c22d0 Answer

Correct Answer: C

Rationale

Choice C is correct. The second equation of the system can be rewritten as $y = 5x - 8$. Substituting $5x - 8$ for y in the first equation gives $5x - 8 = x^2 + 3x - 7$. This equation can be solved as shown below:

$x^2 + 3x - 7 - 5x + 8 = 0$

$x^2 - 2x + 1 = 0$

$(x - 1)^2 = 0$

$x = 1$

Substituting 1 for x in the equation $y = 5x - 8$ gives $y = -3$. Therefore, $(1, -3)$ is the only solution to the system of equations.

Choice A is incorrect. In the xy-plane, a parabola and a line can intersect at no more than two points. Since the graph of the first equation is a parabola and the graph of the second equation is a line, the system cannot have more than 2 solutions. Choice B is incorrect. There is a single ordered pair (x, y) that satisfies both equations of the system. Choice D is incorrect because the ordered pair $(1, -3)$ satisfies both equations of the system.

Question Difficulty: Hard

Question ID fc3dfa26

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: fc3dfa26

3.19

$$\frac{4x^2}{x^2-9}-\frac{2x}{x+3}=\frac{1}{x-3}$$

What value of x satisfies the equation above?

A. -3

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 3

ID: fc3dfa26 Answer

Correct Answer: C

Rationale

Choice C is correct. Each fraction in the given equation can be expressed with the common denominator x^2-9. Multiplying $\frac{2x}{x+3}$ by $\frac{x-3}{x-3}$ yields $\frac{2x^2-6x}{x^2-9}$, and multiplying $\frac{1}{x-3}$ by $\frac{x+3}{x+3}$ yields $\frac{x+3}{x^2-9}$. Therefore, the given equation can be written as $\frac{4x^2}{x^2-9}-\frac{2x^2-6x}{x^2-9}=\frac{x+3}{x^2-9}$. Multiplying each fraction by the denominator results in the equation $4x^2-(2x^2-6x)=x+3$, or $2x^2+6x=x+3$. This equation can be solved by setting a quadratic expression equal to 0, then solving for x. Subtracting $x+3$ from both sides of this equation yields $2x^2+5x-3=0$. The expression $2x^2+5x-3$ can be factored, resulting in the equation $(2x-1)(x+3)=0$. By the zero product property, $2x-1=0$ or $x+3=0$. To solve for x in $2x-1=0$, 1 can be added to both sides of the equation, resulting in $2x=1$. Dividing both sides of this equation by 2 results in $x=\frac{1}{2}$. Solving for x in $x+3=0$ yields $x=-3$. However, this value of x would result in the second fraction of the original equation having a denominator of 0. Therefore, $x = -3$ is an extraneous solution. Thus, the only value of x that satisfies the given equation is $x = \frac{1}{2}$.

Choice A is incorrect and may result from solving $x + 3 = 0$ but not realizing that this solution is extraneous because it would result in a denominator of 0 in the second fraction. Choice B is incorrect and may result from a sign error when solving $2x - 1 = 0$ for x. Choice D is incorrect and may result from a calculation error.

Question Difficulty: Hard

Question ID 58b109d4

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	■■■

ID: 58b109d4

3.20

$$x^2 + y + 7 = 7$$
$$20x + 100 - y = 0$$

The solution to the given system of equations is (x, y). What is the value of x?

ID: 58b109d4 Answer

Correct Answer: -10

Rationale

The correct answer is -10. Adding y to both sides of the second equation in the given system yields $20x + 100 = y$. Substituting $20x + 100$ for y in the first equation in the given system yields $x^2 + 20x + 100 + 7 = 7$. Subtracting 7 from both sides of this equation yields $x^2 + 20x + 100 = 0$. Factoring the left-hand side of this equation yields $(x + 10)(x + 10) = 0$, or $(x + 10)^2 = 0$. Taking the square root of both sides of this equation yields $x + 10 = 0$. Subtracting 10 from both sides of this equation yields $x = -10$. Therefore, the value of x is -10.

Question Difficulty: Hard

Question ID 7028c74f

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: 7028c74f

3.21

$5(x+7)=15(x-17)(x+7)$ What is the sum of the solutions to the given equation?

ID: 7028c74f Answer

Correct Answer: 10.33, 31/3

Rationale

The correct answer is $\frac{31}{3}$. Subtracting $5x + 7$ from each side of the given equation yields $0 = 15x - 17x + 7 - 5x + 7$. Since $5x + 7$ is a common factor of each of the terms on the right-hand side of this equation, it can be rewritten as $0 = 5x + 73x - 17 - 1$. This is equivalent to $0 = 5x + 73x - 51 - 1$, or $0 = 5x + 73x - 52$. Dividing both sides of this equation by 5 yields $0 = x + 73x - 52$. Since a product of two factors is equal to 0 if and only if at least one of the factors is 0, either $x + 7 = 0$ or $3x - 52 = 0$. Subtracting 7 from both sides of the equation $x + 7 = 0$ yields $x = -7$. Adding 52 to both sides of the equation $3x - 52 = 0$ yields $3x = 52$. Dividing both sides of this equation by 3 yields $x = \frac{52}{3}$. Therefore, the solutions to the given equation are -7 and $\frac{52}{3}$. It follows that the sum of the solutions to the given equation is $-7 + \frac{52}{3}$, which is equivalent to $-\frac{21}{3} + \frac{52}{3}$, or $\frac{31}{3}$. Note that 31/3 and 10.33 are examples of ways to enter a correct answer.

Question Difficulty: Hard

Question ID e11294f9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear equations in one variable and systems of equations in two variables	

ID: e11294f9

3.22

The solutions to $x^2 + 6x + 7 = 0$ are r and s, where $r < s$. The solutions to $x^2 + 8x + 8 = 0$ are t and u, where $t < u$. The solutions to $x^2 + 14x + c = 0$, where c is a constant, are $r + t$ and $s + u$. What is the value of c?

ID: e11294f9 Answer

Correct Answer: 31

Rationale

The correct answer is 31. Subtracting 7 from both sides of the equation $x^2 + 6x + 7 = 0$ yields $x^2 + 6x = -7$. To complete the square, adding $\frac{6}{2}^2$, or 3^2, to both sides of this equation yields $x^2 + 6x + 3^2 = -7 + 3^2$, or $x + 3^2 = 2$. Taking the square root of both sides of this equation yields $x + 3 = \pm\sqrt{2}$. Subtracting 3 from both sides of this equation yields $x = -3 \pm \sqrt{2}$. Therefore, the solutions r and s to the equation $x^2 + 6x + 7 = 0$ are $-3 - \sqrt{2}$ and $-3 + \sqrt{2}$. Since $r < s$, it follows that $r = -3 - \sqrt{2}$ and $s = -3 + \sqrt{2}$. Subtracting 8 from both sides of the equation $x^2 + 8x + 8 = 0$ yields $x^2 + 8x = -8$. To complete the square, adding $\frac{8}{2}^2$, or 4^2, to both sides of this equation yields $x^2 + 8x + 4^2 = -8 + 4^2$, or $x + 4^2 = 8$. Taking the square root of both sides of this equation yields $x + 4 = \pm\sqrt{8}$, or $x + 4 = \pm 2\sqrt{2}$. Subtracting 4 from both sides of this equation yields $x = -4 \pm 2\sqrt{2}$. Therefore, the solutions t and u to the equation $x^2 + 8x + 8 = 0$ are $-4 - 2\sqrt{2}$ and $-4 + 2\sqrt{2}$. Since $t < u$, it follows that $t = -4 - 2\sqrt{2}$ and $u = -4 + 2\sqrt{2}$. It's given that the solutions to $x^2 + 14x + c = 0$, where c is a constant, are $r + t$ and $s + u$. It follows that this equation can be written as $x - r + tx - s + u = 0$, which is equivalent to $x^2 - r + t + s + ux + r + ts + u = 0$. Therefore, the value of c is $r + ts + u$. Substituting $-3 - \sqrt{2}$ for r, $-4 - 2\sqrt{2}$ for t, $-3 + \sqrt{2}$ for s, and $-4 + 2\sqrt{2}$ for u in this equation yields $-3 - \sqrt{2} + -4 - 2\sqrt{2}$ $-3 + \sqrt{2} + -4 + 2\sqrt{2}$, which is equivalent to $-7 - 3\sqrt{2}$ $-7 + 3\sqrt{2}$, or $-7 -7 - 3\sqrt{2}$ $3\sqrt{2}$, which is equivalent to $49 - 18$, or 31. Therefore, the value of c is 31.

Question Difficulty: Hard